Modeling Forces Generated by Muscles and Tendons

BioE215 Physics-based Simulation of Biological Structures
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Clay Anderson, Scott Delp, Paul Mitiguy
Paul and Jeff may tell you differently, but… without forces, mass is irrelevant!!!

\[
\vec{F} = m \vec{a}
\]

\[
\vec{a} = \frac{\vec{F}}{m} = \frac{\vec{0}}{m} = \vec{0}
\]
Why are muscle forces important?

- Moving (walking, running, waving, …)
- Talking
- Breathing
- Seeing
- Hearing
- Digesting (smooth muscle)
- Pumping (cardiac muscle)
We need muscles to move
Without muscles...
Muscle & tendon properties influence performance

Muscle strength and the rate at which muscles contract are major determinants of running speed.

Kangaroos can run more efficiently at fast speeds than at slow speeds, partly because of the compliance of their tendons.
Joint contact forces are largely due to muscle forces.

Strong quadriceps can lift a small car off the ground.

\[ F_o^M \approx 10,000 \, N \approx 1,000 \, \text{kg} \]

\[ VW \, Bug \approx 1,180 \, \text{kg}. \]

Joint contact forces:
3 * Body Weight during walking
5 * Body Weight during running
Joint contact forces are largely due to muscle forces.

Joint Disease and Joint Replacements

Bone Development

Osteoarthritis, Osteoporosis, Bone loss in space

Piazza & Delp, 2001

Arnold & Delp, 2001
Muscles are the targets of treatments

Rectus Femoris

Non-Operated  Transferred

Stiff-Knee Gait
Connecticut Children’s Medical Center
What we’ll cover today

• Muscles & simulation

• Muscle mechanics (biology)

• Muscle models

\[ F^T = f(a, F^M_o, L^{MT}, \dot{L}^{MT}) \]

\[ \frac{d F^T}{dt} = f(a, F^M_o, L^{MT}, \dot{L}^{MT}) \]
What we’ll cover today

• Muscles & simulation

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• Muscle models

\[
F^T = f(a, F_o^M, L^{MT}, \dot{L}^{MT})
\]

\[
\frac{dF^T}{dt} = f(a, F_o^M, L^{MT}, \dot{L}^{MT})
\]
How movement is generated

- Muscle excitations
- Muscle activations
- Muscle forces
- Joint torques
- Accelerations
Simulation
Activation dynamics

Excitation-Contraction Coupling

\[
\frac{da}{dt} = \frac{x^2 - xa}{\tau_{rise}} + \frac{x - a}{\tau_{fall}}
\]

\(a \approx Ca^{++}\)
Contraction dynamics
Contraction dynamics (Muscle Mechanics)

\[
\frac{d F^T}{dt} = f(a, F^T, L^{MT}, \dot{L}^{MT})
\]
Forward Dynamic Simulation
Musculoskeletal Geometry (SIMM)

Straight Lines

Wrapping
Forward Dynamic Simulation

Motor Control \( \dot{x} \) \rightarrow \text{Activation Dynamics} \( \dot{a} \) \rightarrow \text{Contraction Dynamics} \( \ddot{f} \) \rightarrow \text{Musculoskeletal Geometry} \( \ddot{\ell} \) \rightarrow \text{Skeletal Dynamics} \( \ddot{q} \) \rightarrow \text{Integration} \( \dddot{q} \)
Skeletal Dynamics

10 Rigid Bodies
- mass
- center of mass
- inertia tensor

10 Joints

23 Degrees of Freedom

Equations of Motion

\[
\frac{d^2 \ddot{q}}{dt^2} = \ddot{I}^{-1} \left\{ \ddot{G} + \ddot{C} + \ddot{R} \cdot \ddot{f}_{mt} + \ddot{F}_{\text{ext}} \right\}
\]
Simulating the Musculoskeletal System

Muscle/Tendon Mechanics
(How muscles and tendons generate force)

Simbody
What we’ll cover today

- Muscles & simulation

- Muscle mechanics (biology)

- Muscle models

\[ F^T = f(a, F^M_o, L^M_T, \dot{L}^M_T) \]

\[ \frac{d F^T}{dt} = f(a, F^M_o, L^M_T, \dot{L}^M_T) \]
Muscles actuate movement by developing tension

- Muscles pull, not push.
- Muscles are grouped into antagonistic pairs.
- Tendon connects muscle to bone.
Hierarchical Muscle Structure

- muscle
- fascicle
- fiber = cell
- myofibril
- sarcoplasmic reticulum
- actin
- myosin
- sarcomere, 2-3μm

Adapted from Scientific American, September 2000
Measuring the force-length property of muscle
Force-length property of muscle

Zajac, 1989

Muscle Fiber Length
Measuring the force-velocity property of muscle

McMahon, 1984
Force-velocity property

Zajac, 1989
Muscle architecture

Parallel Fibered

Pennate Muscle
Muscle, tendon, & pennation angle

\[ \alpha = \text{pennation angle} \]

Zajac, 1989
Muscle architecture

\[ F_o^M \propto \text{no. fibers} \]

\[ F_o^M \propto \frac{\text{Volume}}{L_o^M} \equiv \text{PCSA} \]

Physiologic Cross-Sectional Area

\[ F_o^M = \sigma^M \cdot \text{PCSA} \]

\[ \sigma^M \approx 33 \text{ N/cm}^2 \]
Tendon stress-strain properties

Zajac, 1989

\[ \varepsilon^T = \frac{L_T - L_s^T}{L_s^T} \]
What we’ll cover today

• Muscles & simulation

• Muscle mechanics (biology)

• Muscle models

\[ F^T = f(a, F^M_o, L^{MT}, \dot{L}^{MT}) \]

\[ \frac{d F^T}{dt} = f(a, F^M_o, L^{MT}, \dot{L}^{MT}) \]
Classification of muscle models

“The formulation of a satisfactory quantitative representation of contraction dynamics has been elusive.” Zahalak, 1992

Figure 2–1 Classification of muscle models.
Some concepts

**Lumped-Parameter Model**
The distributed properties of all muscle fibers are lumped into a single ideal fiber characterized by parameters appropriate for the whole muscle.

- All fibers are the same length, at the same pennation angle, etc.
- The strength of the muscle is the summed strength of the individual fibers.

![Diagram of muscle and tendon with labeled dimensions and forces](image)
Some concepts

**Line of Action**
Forces act along a line connecting the muscle origin and insertion.
Models in the handout

1. \( F^T \) → CE → \( F^T \)
   - Tendon | Muscle

2. \( F^T \) → CE → PE → \( F^T \)
   - Tendon | Muscle

3. \( F^T \) → CE → PE → \( F^T \)
   - Tendon | Muscle

b. Pennate

\[ L^T - L^M - L^M \cos \alpha \]

\[ F^T \]

\[ \alpha \]

\[ w \]

\[ Tendon \] | Muscle

\[ L^T \] | \( L^M \cos \alpha \)
Some definitions

\[ L = \text{Length} \]
\[ F = \text{Force} \]
\[ T = \text{Tendon} \]
\[ M = \text{Muscle} \]
\[ \alpha = \text{Pennation angle} \]
\[ L^T = \text{Length of tendon} \]
\[ L^M = \text{Length of muscle} \]
\[ L^{MT} = \text{Length of actuator} \]

**CE** = Contractile Element.
Models the active force generating properties of muscle.

**PE** = Parallel Elastic Element
Models the passive force generating properties of muscle.
Five parameters you need to know


- $L_o^M$: Optimal muscle fiber length. Length at which $F^o_M$ is generated.

- $\alpha_o$: Optimal pennation angle. Pennation angle when the fibers are at $L_o^M$.

- $\tilde{V}_{max}^M$: Maximum shortening velocity of muscle normalized by fiber length.

- $L_s^T$: Slack length of tendon. Length at which tendon starts to develop force.

\[ F_o^M = \sigma^M \cdot \frac{Volume}{L_o^M} \]

\[ 2.0 \cdot L_o^M < V_{max}^M < 10 \cdot L_o^M \]

\[ L_s^T = L_{external}^T + L_{internal}^T \]
1a. Simplest

- Assumptions
  - Tendon is inelastic
  - No dependence on length or velocity
  - Parallel fibered
- Parameters
  \[ F^M_o \]
- Time-varying inputs
  \( a \)

\[ 0 \ (\text{off}) \leq a \leq 1.0 \ (\text{fully on}) \]

\[ F^T = F^{CE} = a(t) \cdot F^M_o \]
1b. Simplest with pennate fibers

- Assumptions
  - Tendon is inelastic
  - No dependence on length or velocity
  - Pennate

- Parameters

\[ F^M_o \quad L^M_o \quad \alpha_o \quad L^T_s \]

- Time-varying inputs

\[ a \quad L^{MT} \]

\[ F^T = a(t) \cdot F^M_o \cdot \cos \alpha \]

But, \( \alpha \) changes with the length of the muscle!
Getting actuator length

\[ L^{MT} = \text{path distance from muscle origin to insertion} \]
1b. Simplest with pennate fibers

1. \[ L^{MT} = L^T_s + L^M \cos \alpha \]

2. \[ w = L^M \sin \alpha = L^M_o \sin \alpha_o \]

Width is assumed to be constant.

Some algebra and trig…

\[
\cos \alpha = \sqrt{\frac{\left( \frac{L^{MT} - L^T_s}{w} \right)^2}{1 + \left( \frac{L^{MT} - L^T_s}{w} \right)^2}}
\]

\[ F^T = a(t) \cdot F^M_o \cdot \cos \alpha \]
2a. Force-length-velocity properties and inelastic tendon

- **Assumptions**
  - Tendon is inelastic
  - Dependence on length or velocity
  - Parallel fibered

- **Parameters**
  \[ F^o_M, L^M, L^T_s, \tilde{V}_{\text{max}} \]

- **Time-varying inputs**
  \[ L^{MT}, \dot{L}^{MT} \]

\[
F^T = F^{CE} + F^{PE} \\
F^T = F^{CE} (L^M, \dot{L}^M) + F^{PE} (L^M)
\]
2a. Force-length-velocity properties and inelastic tendon

\[ F^{PE}(L^M) = F_o^M \cdot 3 \cdot 10^4 \cdot \exp[6 \cdot (\frac{L^M}{L_o^M} - 3.2)] \]
2a. Force-length-velocity properties and inelastic tendon

\[ F^{CE} = a(t) \cdot F^M_o \cdot \tilde{F}^M_L (L^M) \cdot \tilde{F}^M_V (\dot{L}^M) \]

\[ \tilde{F}^M_L (L^M) = \exp[17.33 \cdot \left(\frac{L^M}{L_o^M} - 1.0\right)^3] \]

\[ \tilde{F}^M_V (\dot{L}^M) = 1.8 - \frac{1.8}{1.0 + \exp\left[0.04 \cdot \frac{\dot{L}^M}{\bar{V}_{\text{max}} \cdot L_o^M}\right]} \]
2a. Force-length-velocity properties and inelastic tendon

\[ L^M = L^{MT} - L^T_s \]

\[ \dot{L}^M = \dot{L}^{MT} \]
2a. Force-length-velocity properties and inelastic tendon

\[ F^{PE}(L^M) = F_o^M \cdot 3 \cdot 10^4 \cdot \exp[6 \cdot (\frac{L^M}{L_o^M} - 3.2)] \]

\[ \tilde{F}_L^M(L^M) = \exp[17.33 \cdot \left| \frac{L^M}{L_o^M} - 1.0 \right|^3] \]

Force / \(F_o^M\)

Zajac, 1989
2a. Force-length-velocity properties and inelastic tendon

\[ \tilde{F}_v^M (\dot{L}^M) = 1.8 - \frac{1.8}{1.0 + \exp\left( \frac{0.04 - \dot{L}^M}{\dot{V}_{\text{max}} \cdot L_o^M} \right) } \]

\[ \frac{\dot{L}^M}{\dot{V}_{\text{max}} \cdot L_o^M} \]

Zajac, 1989
3a. Force-length-velocity properties and *elastic* tendon

- **Assumptions**
  - Tendon is elastic
  - Dependence on length or velocity
  - Parallel fibered

- **Parameters**
  \[
  F^M_o, L^M_o, L^T_s, \tilde{V}_{max}
  \]

- **Time-varying inputs**
  \[
  a, L^{MT}, \dot{L}^{MT}
  \]

\[
L^{MT} = L^T + L^M
\]

\[
\dot{L}^{MT} = \dot{L}^T + \dot{L}^M
\]

A closed-form expression for \(F^T\) is generally not possible.
3a. Force-length-velocity properties and *elastic* tendon

\[ F^T = F^T(t = 0) + \int_{t=0}^{t} \dot{F}^T \, dt \]

One last thing about tendon…

\[ k^T \approx \frac{37.5 \cdot F^M_o}{L^T_s} \]

\[ F^T = k^T \cdot (L^T - L_s^T) \]

\[ \dot{F}^T = k^T \dot{L}^T \]

\[ \dot{F}^T = k^T \left( \dot{L}^{MT} - \dot{L}^M \right) \]
Where to get model parameters... the literature.

Garner & Pandy (2001)

MUSCULOSKELETAL MODEL OF THE UPPER LIMB

TABLE I  Architectural properties estimated for each musculotendon actuator in the upper-limb model: volume (Vol), maximum musculotendon length ($L_{\text{max}}^{MT}$), minimum musculotendon length ($L_{\text{min}}^{MT}$), physiological cross-sectional area (PCSA), optimal muscle-fiber length ($L_{o}^{M}$), tendon slack length ($L_{s}^{T}$), maximum isometric muscle force ($F_{o}^{M}$), and muscle pennation angle ($\alpha$). Physiological cross-sectional area was defined as the ratio of muscle volume to optimal muscle-fiber length [35]

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Abbr.</th>
<th>Vol (cm$^3$)</th>
<th>$L_{\text{max}}^{MT}$ (cm)</th>
<th>$L_{\text{min}}^{MT}$ (cm)</th>
<th>PCSA (cm$^2$)</th>
<th>$L_{o}^{M}$ (cm)</th>
<th>$L_{s}^{T}$ (cm)</th>
<th>$F_{o}^{M}$ (N)</th>
<th>$\alpha$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>subclavius</td>
<td>SBCL</td>
<td>8.80</td>
<td>7.15</td>
<td>6.28</td>
<td>4.36</td>
<td>2.02</td>
<td>5.07</td>
<td>144.02</td>
<td>0.00</td>
</tr>
<tr>
<td>serratus anterior (superior)</td>
<td>SRAs</td>
<td>92.20</td>
<td>12.24</td>
<td>5.84</td>
<td>8.12</td>
<td>11.35</td>
<td>0.27</td>
<td>268.05</td>
<td>0.00</td>
</tr>
<tr>
<td>serratus anterior (middle)</td>
<td>SRAm</td>
<td>71.71</td>
<td>19.32</td>
<td>11.23</td>
<td>4.00</td>
<td>17.91</td>
<td>0.75</td>
<td>132.12</td>
<td>0.00</td>
</tr>
<tr>
<td>serratus anterior (inferior)</td>
<td>SRAi</td>
<td>194.65</td>
<td>24.89</td>
<td>13.43</td>
<td>8.41</td>
<td>23.15</td>
<td>0.01</td>
<td>277.51</td>
<td>0.00</td>
</tr>
<tr>
<td>trapezius (Clav-Co)</td>
<td>TRPc</td>
<td>116.23</td>
<td>20.12</td>
<td>9.46</td>
<td>6.24</td>
<td>18.62</td>
<td>0.48</td>
<td>205.05</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Summary

• Quantifying muscle and tendon force is important for understanding
  – Performance
  – Bone and joint mechanics, development, and disease
  – Movement disorders.

• The forces that muscles generate depend nonlinearly on length and shortening velocity.

• Lumped-parameter models vary in complexity
  – The simplest can be developed using closed-form expressions
  – More complex models require ordinary differential equations

• “Reasonable” models can be formulated based on a few parameters
  \[ F_o^M \quad L_o^M \quad \alpha_o \quad \tilde{V}^{M}_{\text{max}} \quad L_s^T \]
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Scaling
Muscle shortens as the proteins slide past each other

QuickTime™ and a Cinepak decompressor are needed to see this picture.

[http://www.sci.sdsu.edu/movies/actin_myosin.gif.html]
Muscle architecture

Sartorius has long, parallel fibers and very little tendon

Gastrocnemius has short, pennate fibers and a long tendon
Hierarchical Muscle Structure

- muscle
- fascicle
- fiber = cell
- myofibril
- sarcomere, 2-3 μm
- actin
- myosin
- sarcoplasmic reticulum
- filaments

Adapted from Scientific American, September 2000
Sarcomere
Actin-myosin cross-bridges
Tension developed by a sarcomere
Simulations Generated with CMC

Each generated with less than 10 minutes of CPU time.
Simulated Treatments

Botulinum Toxin Injection

RF Excitation

double swing

gait cycle

reduces both
hip flexion
knee extension
moments

botox

RF Transfers

preserve hip flexion
moments

eliminates
knee extension
moment

IT Band

Sartorius

generates
knee flexion
moment
Simulations can help us understand muscle function
Simulations can help us understand muscle function