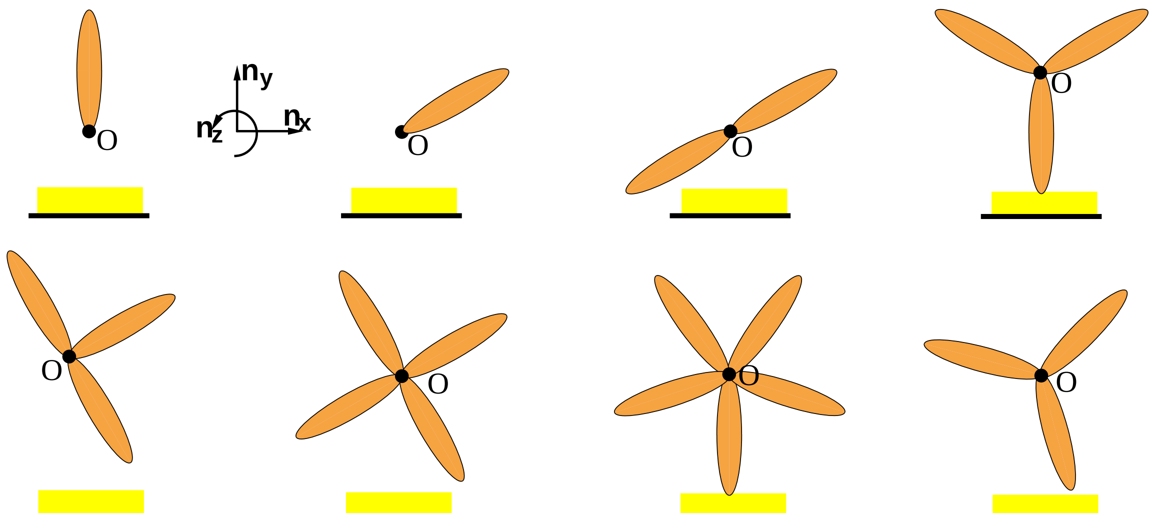
Lab 4: Mass distribution of rigid bodies in Simbody

4.1 Lab: Conceptual example of products of inertia

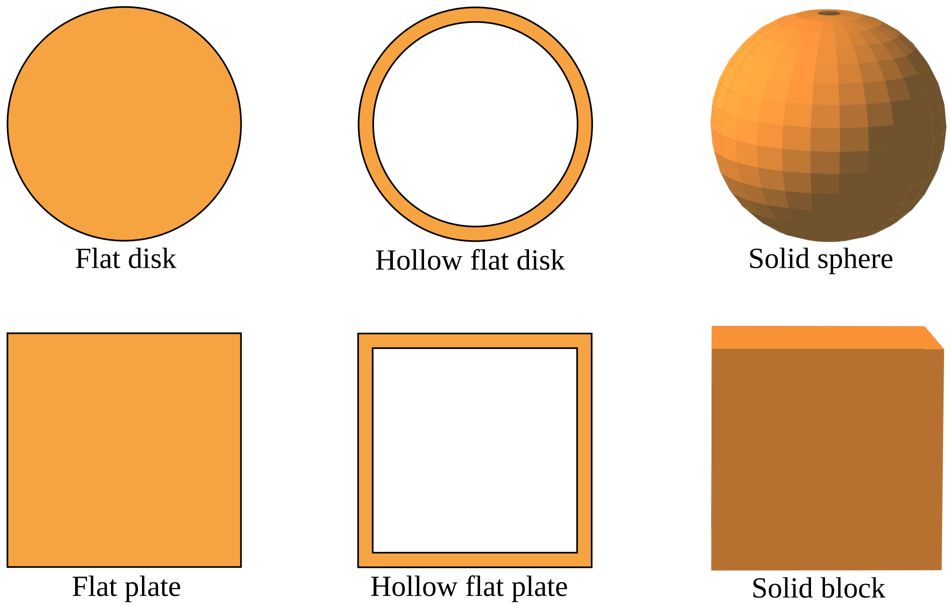
The figure below shows six objects, each with a uniform density. For each object, consider $I_{\mathbf{n}_x \mathbf{n}_y}$, the product of inertia of the object for lines that pass through point O and are parallel to $\mathbf{n}_x$ and $\mathbf{n}_y$. Below each object, mark whether the product of inertia is ***negative***, ***zero***, or ***positive***.



[0]Last updated May 7, 2007 by Paul Mitiguy.

4.2 Lab: Conceptual example of moments of inertia

Each object below has a uniform density and an equal mass. After identifying the mass center of each figure with an , answer the following questions about I_{zz}, the moment of inertia of each object about the line that passes through its mass center and is perpendicular to the plane of the paper.[1]



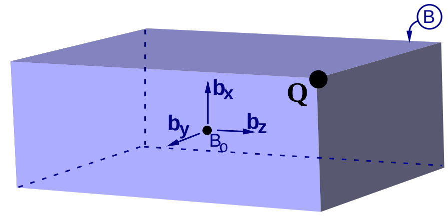
- Consider the first row of objects. The **flat disk/hollow disk/solid sphere** has the **largest** value of I_{zz}, whereas the **flat disk/hollow disk/solid sphere** has the **smallest** value of I_{zz}.

- Consider the second row of objects. The **flat plate/hollow plate/solid block** has the **largest** value of I_{zz}, whereas the ________ and ________ have **equal** values of I_{zz}.

- Consider all the objects in both rows. The ________ has the **largest** value of I_{zz}, whereas the ________ has the **smallest** value of I_{zz}.

[1]The conceptual questions about moment of inertia are answered by noting that I_{zz} varies with $m*d^2$. The objects with the highest concentration of mass furthest away from the mass center have the largest moment of inertia. The objects with the highest concentration of mass closest to the mass center have the smallest moment of inertia.

4.3 Lab: Basic inertia calculations



The figure to the right shows a particle Q of mass m whose position from the centroid B_o of a rectangular parallelepiped B is

$$\mathbf{r}^{Q/B_o} = x\,\mathbf{b}_x + y\,\mathbf{b}_y + z\,\mathbf{b}_z$$

1. Use the definition of the inertia dyadic of a particle to find the ***inertia dyadic*** of Q about B_o.
   Result:

$$\begin{aligned}\underline{\mathbf{I}}^{Q/B_o} = \; & \boxed{}\,\mathbf{b}_x\mathbf{b}_x + \boxed{}\,\mathbf{b}_x\mathbf{b}_y + \boxed{}\,\mathbf{b}_x\mathbf{b}_z \\ + \; & \boxed{}\,\mathbf{b}_y\mathbf{b}_x + \boxed{}\,\mathbf{b}_y\mathbf{b}_y + \boxed{}\,\mathbf{b}_y\mathbf{b}_z \\ + \; & \boxed{}\,\mathbf{b}_z\mathbf{b}_x + \boxed{}\,\mathbf{b}_z\mathbf{b}_y + \boxed{}\,\mathbf{b}_z\mathbf{b}_z\end{aligned}$$

2. Find the ***inertia matrix*** of Q about B_o for $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$.
   Result:

$$I^{Q/B_o}_{\mathbf{b}_{xyz}} = \begin{bmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{bmatrix}_{\mathbf{b}_{xyz}}$$

3. I^{Q/B_o}_{xx} (the moment of inertia of Q about B_o for $\mathbf{b}_x$) is equal to the mass of Q multiplied by the square of the distance of Q from B_o. **True/False**.

4. I^{Q/B_o}_{xy} (the product of inertia of Q about B_o for $\mathbf{b}_x$ and $\mathbf{b}_y$) is equal to the mass of Q multiplied by the square of the distance of Q from the $\mathbf{b}_x\,\mathbf{b}_y$ plane that passes through B_o. **True/False**.

5. Determine the ***maximum moment of inertia*** of Q about B_o in terms of m, x, y, and z,
   Result:

$$I^{Q/B_o}_{\text{max}} = m\,(\boxed{})$$

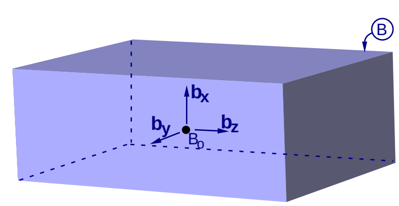
6. Determine the ***minimum moment of inertia*** of Q about B_o and determine a unit vector $\mathbf{u}$ that is parallel to the ***minimum principal axes*** in terms of x, y, and z.
   Result:

$$I^{Q/B_o}_{\text{min}} = \boxed{} \qquad \mathbf{u} = \boxed{}$$

7. Calculate $I^{Q/B_o}_{\mathbf{b}_{xyz}}$ (in units of $\text{kg} * \text{m}^2$) when $m = 1$ kg and the rectangular parallelepiped is 2 m high, 4 m deep, and 6 m wide and Q is located at the top, front, right corner.
   Result:

$$I^{Q/B_o}_{\mathbf{b}_{xyz}} = \begin{bmatrix} 13 & -2 & -3 \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{bmatrix}_{\mathbf{b}_{xyz}}$$

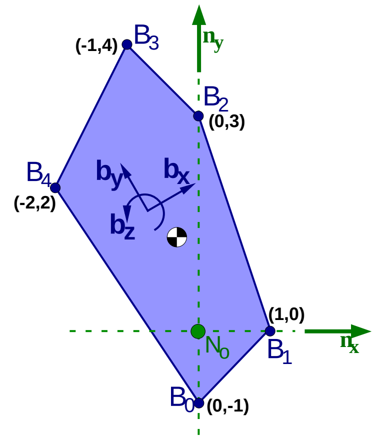
8. If there are other locations on the boundary of the rectangular parallelepiped that have the same $I^{Q/B_o}_{\mathbf{b}_{xyz}}$ draw them on the figure to the right.



4.4 Lab: Mass distribution and bounding rectangle for a polygon

The figure to the right shows a polygon B in a plane N whose vertices are numbered $B_0 \ldots B_4$. Right-handed, orthogonal, unit vectors $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ are fixed in N with $\mathbf{n}_x$ horizontally right, $\mathbf{n}_y$ vertically upward, and $\mathbf{n}_z$ perpendicular to the plane containing B. A second set of orthogonal unit vectors $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ are oriented by initially aligning $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ with $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ and then subjecting $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ to a right-handed rotation characterized by $\theta\,\mathbf{b}_z$.

Note: The orientation of B is not changed when $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ are rotated.



1. Visually determine x_{min} (the minimum $\mathbf{n}_x$ measure of all points on polygon B) and the associated point at which x_{min} occurs. Similarly, determine x_{max}, y_{min}, and y_{max}.
   Result:

Quantity	Description	Value	Associated point
x_{min}	Minimum $\mathbf{n}_x$ measure of B	-2	B_4
x_{max}	Maximum $\mathbf{n}_y$ measure of B	1	B_1
y_{min}	Minimum $\mathbf{n}_x$ measure of B	-1	B_0
y_{max}	Maximum $\mathbf{n}_y$ measure of B	4	B_3

2. Determine the area and centroid of the bounding rectangle whose sides are parallel to $\mathbf{n}_x$ and $\mathbf{n}_y$.
   Result:

   Area = 15 Centroid location = (-0.5, 1.5)

3. Submit a C^{++} program named `PolygonMassDistributionProperties.cpp` that varies θ from 0° to 90° and that prints out results for each value of θ in the file `PolygonMassDistributionResults.txt` in the following format:

```
theta=    0.00000E+000
xMin =   -2.00000E+000   xMax =    1.00000E+000
yMin =   -1.00000E+000   yMax =    4.00000E+000
Bounding rectangle area:   1.50000E+001
```

The prototype for the new function that you need to create and use is

```
void  UpdateBoundingBoxDimension( const Vec3 &vertexPosition, const Vec3 &xUnitVector,
                                  const Vec3 &yUnitVector,    const Vec3 &zUnitVector,
                                  Vec6 &boundingBox,          bool firstCallToFunction );
```

Use this program to determine x_{min} (the minimum $\mathbf{b}_x$ measure of all points on polygon B) when $\theta = 30°$. Similarly, determine x_{max}, y_{min}, and y_{max}. Lastly calculate the area of the bounding rectangle whose sides are parallel to $\mathbf{b}_x$ and $\mathbf{b}_y$ when $\theta = 30°$.
Result:

Quantity	Description	Value
x_{min}	Minimum $\mathbf{b}_x$ measure of B	-2
x_{max}	Maximum $\mathbf{b}_y$ measure of B	1
y_{min}	Minimum $\mathbf{b}_x$ measure of B	-1
y_{max}	Maximum $\mathbf{b}_y$ measure of B	4

Area =

4. Determine the area of the smallest bounding rectangle and the associated value of θ.
Result:
Minimum area of all bounding rectangles $= 10.5226$ Corresponding value of $\theta = \boxed{}^\circ$

5. Optional**: Assume B has a uniform density of $1\ \frac{\text{kg}}{\text{m}^2}$, calculate the area of B, mass of B, the position vector of B_{cm} (the mass center of B) from N_o, the ***inertia matrix*** of B about B_{cm} for $\mathbf{n}_\text{x}$, $\mathbf{n}_\text{y}$, $\mathbf{n}_\text{z}$.
Result:

$$\text{Area} = 7.5 \qquad m^B = 7.5 \qquad \mathbf{r}^{B_{cm}/N_o} = \text{-}0.4667\,\mathbf{n}_\text{x} + \boxed{}\,\mathbf{n}_\text{y}$$

$$I^{B/B_{cm}} = \begin{bmatrix} 9.457407 & 2.830556 & 0 \\ 2.830556 & 3.116667 & 0 \\ 0 & 0 & 12.57407 \end{bmatrix}_{\mathbf{n}_\text{xyz}}$$

6. What method does one use to calculate a polygon's area, mass, center of mass, and inertia properties.
Result:

$\boxed{}$

7. Determine the ***principal moments of inertia***, i.e., the principal ***minimum moment of inertia***, ***intermediate moment of inertia***, and ***maximum moment of inertia***.
Result:

$$I^{B/B_{cm}}_\text{min} = 2.036944 \qquad I^{B/B_{cm}}_\text{intermediate} = \boxed{} \qquad I^{B/B_{cm}}_\text{max} = \boxed{}$$

8. Express $\mathbf{b}_\text{x}$ in terms of $\mathbf{n}_\text{x}$, $\mathbf{n}_\text{y}$, $\mathbf{n}_\text{z}$ when $\mathbf{b}_\text{x}$ is parallel to the intermediate principal axis of B, and determine the angle θ between $\mathbf{b}_\text{x}$ and $\mathbf{n}_\text{x}$. (Hint: Eigenvectors.)
Result:

$$\mathbf{b}_\text{x} = 0.9343321\,\mathbf{n}_\text{x} + \boxed{}\,\mathbf{n}_\text{y} \qquad \theta = \boxed{}^\circ$$