Energy aspects for elastic and viscous shoe soles and playing surfaces

BENNO M. NIGG and MICHAEL ANTON

Human Performance Laboratory,
The University of Calgary,
Calgary, Alberta, CANADA T2N 1N4

ABSTRACT

NIGG, B. M. and M. ANTON. Energy aspects for elastic and viscous shoe soles and playing surfaces. *Med. Sci. Sports Exerc.*, Vol. 27, No. 1, pp. 92–97, 1995. The purpose of this project was to determine the effect of changes in stiffness and viscosity of the foot ground interface on the work performed during locomotion. The estimation of the work during locomotion was derived from a mathematical two segment model, representing the foot and the rest of the body. The typical passive elements between the foot and the rest of the body were replaced by a strategic formulation of how a resultant force, F, representing the net effect of all the muscles between the foot and the rest of the body, has to evolve over time in a running situation. The calculations were performed under the assumption that the force F is selected so that the mechanical work performed by F is minimal. The estimations of the work required during a step cycle is generally higher for softer than for harder springs and for low damping compared with high damping. The model calculations demonstrate that specific combinations of material properties may be advantageous or disadvantageous from an energy point of view.

ENERGY, LOCOMOTION, SPORT SHOES, LOSS OF ENERGY, VISCO-ELASTIC, STIFFNESS

Ground reaction forces in running consist of an impact and an active component (6,9,13). Three materials are used to "cushion" the landing of the heel during running, the material of the running surface, the midsole material of the shoe, and the soft tissue material of the heel. They have elastic and viscous properties. It may be speculated that the combination of elastic and viscous elements in the surface-shoe-heel material is of importance for the economy of running (3) and/or that the dominant elastic behavior forces specific movement patterns onto the overall system (e.g., vibrations) that can be disadvantageous for the running economy. To discuss these speculations, a theoretical model has been developed. The purpose of this model is to investigate how the work requirements in running depend on different viscoelastic characteristics of the surface-shoe-heel interface.

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Previous models, which represent the combined effect of all the leg and hip muscles involved in running by passive elements such as springs and dampers (5,12) were found to be inadequate for the purpose of this model. The very fact that serious running is quite strenuous seems to contradict the approach of modeling muscles by energy conserving mechanical elements like springs. The total mechanical energy content in a system composed exclusively of masses and springs remains constant over time. Only the relative amount of kinetic and potential energy changes. The question of how much work is performed in such a system is, therefore, meaningless and spring-mass systems are not suited to respond to the purpose of this investigation. Additionally, including damper elements in the system has the effect of decreasing the mechanical energy content over time. The lost energy cannot be regained for lack of active components in these models. As a result, models consisting exclusively of masses, springs, and dampers cannot describe energy aspects of a sustained running motion.

The presented model (4) represents an attempt to replace the passive mechanical elements (spring and dampers) between the foot and the rest of the body by a strategic formulation of how a resultant force, representing the net effect of all the muscles between the foot and the rest of the body has to evolve over time in a running situation. The derived model is then used to study work requirements for changing surface-shoe-heel characteristics.

Assumptions

1) The human body is subdivided into two masses, one mass, m_1, representing the foot of the support leg and another mass, m, representing the rest of the body (Fig. 1).

2) Effects of all extremities, except the support leg, on the upper body are neglected.

3) The horizontal velocity of the upper body is assumed to be constant. Consequently, the model neglects

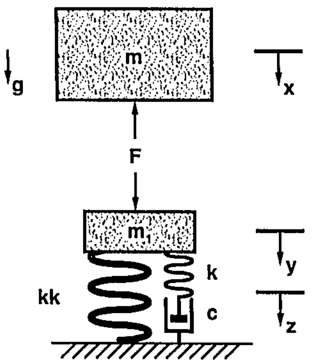

Figure 1—Illustration of the mechanical model used in this example.

the movement in the horizontal direction, considering only the vertical movement. The model is, therefore, one-dimensional.

4) A spring-damper combination, k, kk, and c, represents the combined material properties of the surface, the shoe midsole and the human heel.

5) The ground is assumed to be rigid.

6) A force, F, acts between the upper body and the foot.

7) During the flight phase the runner's body moves freely in the conservative gravitational force field. For that time interval the total mechanical energy (sum of kinetic and potential energy) is assumed (as a first approximation) to be constant.

8) The mathematical analysis is limited to the stance phase.

9) The force F between the foot and the upper body develops in such a way as to minimize the work it performs in bringing the upper body from touch-down to take-off. In its mathematical expression this leads to an open loop optimal control problem.

10) The muscles that generate the force F are assumed to be incapable of energy storage. Therefore, they can perform work for upward and downward movements of the mass m.

Model

The equations of motion for the illustrated set-up are:

$$\ddot{x} = m \cdot g - F$$

$$\ddot{y} = m_1 \cdot g + F - (kk + k) \cdot y + k \cdot z$$

$$\dot{z} = \frac{k}{c}(y - z)$$

where

g = acceleration due to gravity

x = coordinate describing the movement of mass m

y = coordinate describing movement of mass m_1

y = coordinate describing the movement of damper c

The question will be solved as an optimization problem. In this process the equations of motion act as constraints to the optimization problem.

$$\int_0^{\Delta t} [\,|F(\dot{x} - \dot{y})| + a \cdot F^2 + b \cdot f(\dot{F})]dt \rightarrow \min$$

where F is the muscle force which is a function of time and

$$\begin{aligned} f(\dot{F}) &= (\dot{F} - \dot{F}_{max})^2 && \text{if} \quad \dot{F} > \dot{F}_{max} \\ f(\dot{F}) &= 0 && \text{if} \quad \dot{F}_{min} \leq \dot{F} \leq \dot{F}_{max} \\ f(\dot{F}) &= (\dot{F} - \dot{F}_{min})^2 && \text{if} \quad \dot{F} < \dot{F}_{min} \end{aligned}$$

The first term under the integral represents the work performed by the force F.

$$\int_0^{\Delta t} F(\dot{x} - \dot{y})dt \mathrel{\hat{=}} \text{work}$$

The second term under the integral represents the fact that the physiological cost to the system is higher at higher than at lower force levels. The third term under the integral provides a limit for the rate of force increase and decrease, dF/dt. The second time derivative of F is the unknown function which will be determined by the optimization process. The second derivative of F was chosen to be the unknown in order to be able to specify boundary conditions for F and dF/dt. The factors a and b allow to adjust the relative importance of the second and third term under the integral with respect to the first term. The ground reaction force, F_G, is given by the equation

$$F_G = (kk + k) \cdot y - k \cdot z$$

Pontryagin's maximum principle is applied to solve the optimization problem. The first term under the integral supplies the work required for a one-step cycle once the solution to the optimization process is substituted.

Input into the Model

The model was initially tested by using the following inputs:

$\Delta t_1 = 0.1$ s = contact time for "running"

$\Delta t_2 = 0.6$ s = contact time for "walking"

$m = 70$ kg

m_1 7.5 kg

$kk = 2.5 \cdot 10^5$ N/m

$k = 2.5 \cdot 10^5$ N/m

$c = 8.4 \cdot 10^3$ kg/s

$\dot{F}_{max} = +7.5 \cdot 10^4$ N/s

$\dot{F}_{min} = -7.5 \cdot 10^4$ N/s

The stiffness and damping values chosen provide for a stiff and critically dampened foot-surface interface.

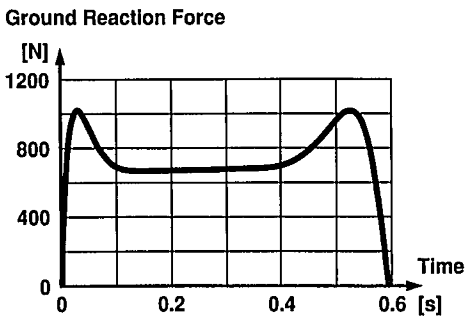

Figure 2—Predicted ground reaction force, F_G, for a contact time of 0.1 s (running).

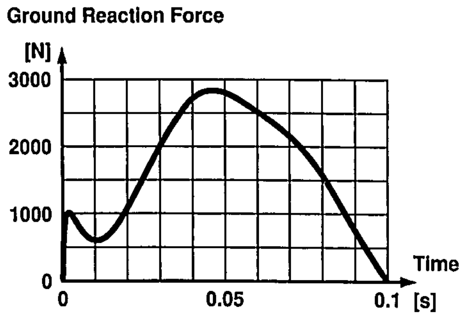

Figure 3—Predicted ground reaction force, F_G, for a contact time of 0.6 s (walking).

Additionally initial and terminal boundary conditions were chosen:

for t = 0 s

$$x = 0 \text{ m}$$
$$y = 0 \text{ m}$$
$$z = 0 \text{ m}$$
$$\dot{x} = +6 \text{ m/s}$$
$$\dot{y} = +6 \text{ m/s}$$
$$\dot{z} = +6 \text{ m/s}$$
$$F = 0 \text{ N}$$
$$F_G = 0 \text{ N}$$

for $t = \Delta t_i$ (0.1 s or 0.6 s)

$$\dot{x} = -0.6 \text{ m/s}$$
$$x - y = 0 \text{ m}$$
$$F_G = 0 \text{ N}$$

The second terminal boundary condition, $x - y = 0$ m, stipulates that the length of the leg at take-off is the same as at touch-down. However, take-off may occur with the masses m and m1 being located at different heights than at touch-down.

Results of the Model and Discussion

The output of the model consists of the ground reaction force, F_G, and the force F acting between the upper body and the foot. The estimation of the ground reaction force is not needed since it is possible to measure the ground reaction force experimentally. However, the ground reaction force can be used to evaluate (validate) the model. If the ground reaction force predicted by the model and the actual ground reaction force are similar, the confidence in the other results is increased. Consequently, the predicted ground reaction forces are compared in a first step with the experimentally determined ground reaction forces. Figure 2 shows the predicted ground reaction force for ''running'' (contact time = 0.1 s). The estimated ground reaction force shows some similarity with the experimentally determined ground reaction force as reported earlier (6,13). It shows the initial impact force peak and the subsequent active force peak.

Figure 3 shows the predicted ground reaction force for walking. The general characteristics for ground reaction forces for walking are present in this predicted force time curve. The curve has the camel-like shape of typical walking curves.

The two comparisons between predicted and experimentally determined ground reaction forces show good agreement in shape as well as in magnitude, which may support the credibility of the proposed model.

Note that only very general assumptions were made for the calculation with this model. In addition to some geometrical assumptions (such as landing and take-off speed are the same) the main assumption was that F is selected so that the mechanical work performed by F is minimal. It is certainly interesting that such a simple mechanical system with very basic assumptions produces ground reaction forces that correspond in magnitude and shape to the actual ground reaction forces measured experimentally. Furthermore, it is interesting that the shape of the ground reaction force changes from short to long contact times (by exclusively changing Δt) the same way it changes from running to walking.

The second step in the model calculations estimated the forces, F, between ''foot'' and ''upper body.'' In a first approximation, these forces can be considered as the forces in the ''ankle joint.'' Figure 4 illustrates the force-time diagram for the force F for the contact time of 0.1 s. The peak value of F is about 10% smaller than the peak value of the ground reaction force, F_G, for the same movement (F_{max} = 2500 N and F_{Gmax} = 2800 N) which is due to dynamic effects at the foot level. Additionally, the impact peak in the ground reaction force, which is solely due to dynamic effects at the foot level, is absent in the force curve for F at the ''ankle joint'' level. The general shape of the estimated force-time curve at the ''ankle joint'' corresponds favorably to the estimated ankle joint forces during running (15). The difference in magnitude of the estimated force-time curve at the "ankle joint" could be explained by the absence of co-contrac-

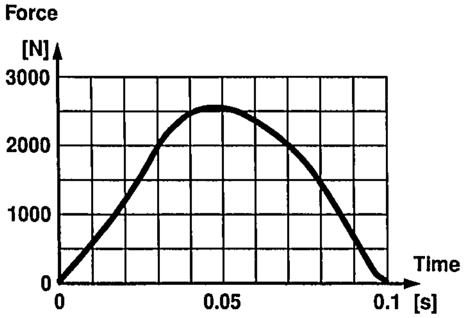

Figure 4—Illustration of the internal force, F, as a function of time for the short contact time, 0.1 s, corresponding to the movement running.

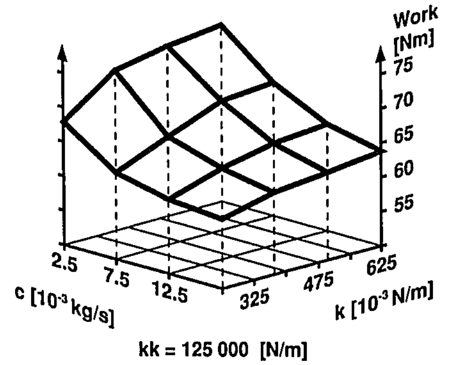

Figure 5—Work required per step cycle for kk = 1.25. 10^5 N/m and variable k and c (case 1).

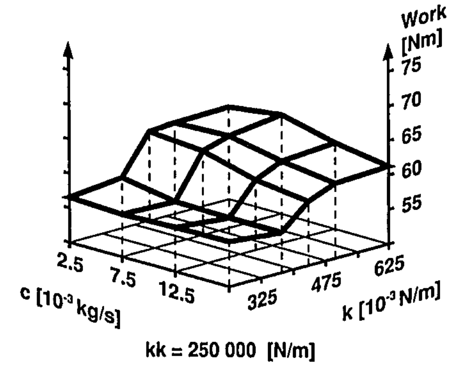

Figure 6—Work required per step cycle for kk = 2.50. 105 N/m and variable k and c (case 2).

tion in our model. The force-time curve, F(t), for the longer contact time of 0.6 s is nearly identical to the ground reaction force curve for the same contact time. Dynamic effects do not have a noticeable effect and the diagram is, therefore, not shown.

In the third step the work performed by F has been calculated for a running situation where $\Delta t = 0.1$ s = constant. This calculation has been done for the following material constants of the elements between the foot and the surface:

kk = $1.25 \cdot 10^5$ N/m (case 1)

kk = $2.5 \cdot 10^5$ N/m (case 2)

k = variable from $2.5 \cdot 10^5$ N/m to $6.25 \cdot 10^5$ N/m

c = variable from $2.5 \cdot 10^3$ kg/s to $17.5 \cdot 10^3$ kg/s

The material constant kk has been chosen in such a way that the maximal static deflection of the foot mass, m_1, under a load of 2500 N was 2 cm for case 1 and 1 cm for case 2. The used values correspond reasonably well to the actual forces and deflections in human movement.

The ranges for k and c have been chosen so that they extend from subcritical damping of a system composed solely of m_1, k, kk, and c to critical damping of a system including m, m_1, k, kk, and c. This range was assumed to cover the actual range of possibilities for running.

The estimations of the performed work during a step cycle (Figs. 5 and 6) indicates that the amount of work required is generally higher for case 1 (the softer spring constant for kk) than for case 2 (the harder spring constant for kk). For case 1, the softer spring kk, the work performed decreases steadily with increasing c and decreasing k. Higher values for c, the damper, have the effect of making the foot-surface interface dynamically stiffer, resulting in a lower damper deflection and consequently in lower damping. Increasing values of k communicate more force to the damper which results in higher damper deflections and higher damping.

Case 2, where the spring stiffness of kk is higher, presents a completely different connection between the material properties of k and c and the work performed. The influence of the damping coefficient, c, is quite small. However, the influence of k becomes more crucial. There is a critical range in k values over which the work requirements change rapidly, whereas they remain fairly constant over the remaining intervals of k and c. This result is, from a practical standpoint, interesting. There is a critical combination of material properties where relatively small changes in material properties are associated with work increases of about 10%.

This example is only a first step in the attempt to determine the effect of various material properties and/or movement changes on the work requirements during running. The model does not provide specific detail about the critical material properties for reduced or increased work requirements. As a matter of fact, one can not really separate the material constants k and c. The required work depends on the combined effect of k and c. However, the model illustrates that specific combinations of material properties may be advantageous or disadvantageous from an energy point of view.

The simple model provides another understanding: the work requirement is not exclusively dependent on how much energy is lost in the damper. Work is performed by the muscles in slowing down the upper body after touchdown and in accelerating it up toward take-off. The amount of work performed is the sum of force multiplied by displacement increments. The functional dependencies of force and displacement on time are interdependent for the problem under investigation and they depend on the visco-elastic foot-shoe-surface interface. They must assume a form that fulfils the task of bringing the mass m from touch-down to lift-off under the given boundary conditions. Note, that even if c was set to zero (which corresponds to the fully elastic condition), the resulting work requirement would still be unequal to zero and would assume different values for different spring constants kk.

There is evidence that part (but only part) of the kinetic energy at touch-down is stored in elastic tissue during the stance phase (2). This does not contradict the approach taken here. Even if the muscles are required to perform only part of the work given by the first term in the integral equation, it would still make sense for this portion to be minimal. However, the muscle tendon units have been assumed in this approach as being unable to store energy. The fact that the theoretically estimated ground reaction force is close to the experimentally determined ground reaction force may suggest that the significance of storage of elastic energy in the muscle-tendon units during running may have to be carefully reconsidered.

The proposed modeling approach determined an integral force, F, which can be considered as the result of all forces produced due to muscle activity, gravity and inertia at the ankle joint level. The initial and terminal boundary conditions chosen were realistic for running. The resultant F_G-time curves as estimated from the model and the F_G-time curves as determined from experiments show good agreement in magnitude and shape. This may suggest that the presented optimal control model for running and its underlying postulate of minimum performed work is acceptable.

The work/energy balance during human locomotion can be influenced (2) by returning energy to the locomotor system (for instance from a sport surface) and/or (3) by conserving energy in the first place (14). Previous research on this question concentrated primarily on the first aspect, the storage and return of energy (2,11,12). Sport shoes and/or playing surfaces for which energy should be stored and returned require substantial deformation (12,14). Additionally, the requirements for a sprint or a marathon run are different. Energy returning constructions have been successfully implemented for indoor tracks (12) but not for outdoor tracks and/or sport shoes. Storage of elastic energy (2,11) in the human body, however, depends on the appropriate use of the phenomenon of resonance, and not the stored elastic energy (7). The resonance characteristics of the lower extremities, for instance, depends among other factors on the mechanical muscle characteristics, an aspect which implicitly has been addressed in the presented model. The results of the present model calculations suggest that the second approach, the conservation of energy, may be at least as important for work considerations in human locomotion as the possible return of energy.

The presented model can and should not be used to predict the specific material properties for a shoe sole or a playing surface to reduce work and to improve performance. It is not specific enough to fulfill this function. However, the results of these model calculations suggest that viscous properties of materials of playing surfaces, shoes and the human heel may be advantageous for the reduction of work performed during locomotion. Support for this statement may be drawn from selected excellent performances of athletes in long distance events running barefoot and the fact that the heel of the human foot shows strong viscous behavior (7).

The result that viscous materials in surface, shoe and/or heel should reduce work requirements in the model calculation indicates that the mechanical properties of the foot ground interface influences the movement of the total system. Movement changes may relate to changes in the geometry of movement during ground contact and/or local vibrations of mass segments. The model does not provide any indication which of these changes occur dominantly. However, changes in the ground reaction force pattern of mass spring damper systems during drop tests suggest that damping of vibrations may play an important role in the reduction of work during locomotion (1). The damper in the model reduces vibrations of the two masses quickly. A pure elastic material, however, would not reduce these vibrations and the force F would have to provide work to reduce these vibrations. This speculation may have an application for the human musculo-skeletal system. It may be possible that muscles of the lower extremities are "tuned" to the input signal in order to reduce soft tissue vibrations (wobbling masses, 10). This tuning, however, may be associated with changes in required physiological work. However, further research is needed to investigate how much of the changes in required work depends on general movement changes and whether or not these changes are associated with damping of vibrations.

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