

Complete Description of the Thelen2003Muscle Model

Chand T. John

One of the standard muscle models used in OpenSim is the Thelen2003Muscle actuator. Unfortunately, to my knowledge, no other paper or document, including the Thelen, 2003 paper describing this muscle model, contains the exact equations used to implement the force-length, force-velocity, and tendon force-strain relationships in the OpenSim source code. Therefore, it is described below.

Muscle Model

In simulation studies, a muscle model is an algorithm transforming muscle activation into muscle force. Here we describe the muscle model used in our simulation, which is a slight modification of the Hill-type muscle model presented in Thelen, 2003. At a given time t in a simulation, the muscle model's inputs are activation $a(t)$ (a real number between 0 and 1 inclusive) and fiber length $l^M(t)$. The muscle model's outputs are the muscle-tendon actuator force $F^{MT}(t)$ and, for the next time step $t + \Delta t$, the time-derivative of activation $\dot{a}(t + \Delta t)$ and the time-derivative of fiber length, i.e., fiber velocity $\dot{l}^M(t + \Delta t)$. Here we will describe how the muscle-tendon actuator force, time-derivative of activation, and fiber velocity are calculated.

Activation dynamics represents how an excitation $u(t)$, a unit-less value between 0 and 1 is transformed into an activation $a(t)$, also a unit-less value between 0 and 1, of a muscle. When a muscle is neurally excited, its activation gradually increases, while if a muscle's excitation decreases, its activation also decreases, albeit at a slower rate than the increase. The time-derivative of activation is calculated as:

$$\dot{a}(t + \Delta t) = \begin{cases} (u(t) - a(t)) / \tau_{act}, & u(t) \geq a(t) \\ (u(t) - a(t)) / \tau_{deact}, & u(t) < a(t) \end{cases},$$

where $\tau_{act} = 10$ ms and $\tau_{deact} = 40$ ms.

Our muscle model consists of two parts: the muscle fibers and the tendon. The muscle fibers may be slanted relative to the tendon at an angle called the pennation angle. The muscle fibers consist of a passive element that produces passive force (i.e., force due to the fibers' inherent stiffness) and a contractile element that produces active force (i.e., force due to activation of the muscle).

Our muscle model contains several constants. Some constants may vary between muscles in our musculoskeletal model of the body while other constants are the same for all muscles. The constants that may vary between muscles are:

- F_0^M , the maximum isometric force of the muscle
- l_0^M , the optimal fiber length of the muscle
- l_s^T , the tendon slack length, the tendon length below which the tendon (and therefore the whole muscle) produces zero force

- α_0 , the pennation angle of the muscle fibers at the optimal fiber length

The constants that are the same across muscles are:

- $\varepsilon_0^M = 0.6$, passive muscle strain due to maximum isometric force
- $k_{toe} = 3$, an exponential shape factor
- $\varepsilon_0^T = 0.033$, tendon strain due to maximum isometric force
- $k_{lin} = 1.712 / \varepsilon_0^T$, a linear shape factor
- $\varepsilon_{toe}^T = 0.609 \varepsilon_0^T$, tendon strain above which tendon force is linear with respect to tendon strain
- $\bar{F}_{toe}^T = 0.333333$, normalized tendon force above which tendon force is linear with respect to tendon strain
- $k^{PE} = 4$, an exponential shape factor for the passive force-length relationship
- $\gamma = 0.5$, a shape factor for the Gaussian active force-length relationship
- $A_f = 0.3$, a shape factor for the force-velocity relationship
- $\bar{F}_{len}^M = 1.8$, maximum normalized muscle force achievable when the fiber is lengthening

A bar over variables representing lengths indicates normalization with respect to the optimal fiber length, e.g., $\bar{l}^M = l^M / l_0^M$ is the “normalized fiber length.” A bar over variables representing forces indicates normalization with respect to the maximum isometric force, e.g., $\bar{F}^{MT} = F^{MT} / F_0^M$ is the “normalized muscle-tendon actuator force.”

The computation of muscle-tendon actuator force proceeds as follows. From the configuration of the musculoskeletal model of the body at time t , we obtain the length of the whole muscle-tendon unit, $l^{MT}(t)$. The tendon length is then calculated:

$$l^T(t) = l^{MT}(t) - l^M \cos \alpha(t).$$

From this, a normalized quantity called the tendon strain is calculated:

$$\varepsilon^T = (\bar{l}^T - \bar{l}_s^T) / \bar{l}_s^T.$$

Then the tendon force is calculated as $F^T(t) = F_0^M \bar{F}^T(\varepsilon^T)$, where

$$\bar{F}^T(\varepsilon^T) = 0.001(1 + \varepsilon^T) + \begin{cases} k_{lin}(\varepsilon^T - \varepsilon_{toe}^T) + \bar{F}_{toe}^T, & \varepsilon^T > \varepsilon_{toe}^T \\ \bar{F}_{toe}^T \frac{e^{k_{toe}\varepsilon^T / \varepsilon_{toe}^T} - 1}{e^{k_{toe}} - 1}, & 0 < \varepsilon^T \leq \varepsilon_{toe}^T \\ 0, & \varepsilon^T \leq 0 \end{cases}$$

represents the normalized tendon force-strain relationship, also known as tendon compliance or tendon stiffness. The extra term $0.001(1 + \varepsilon^T)$ exists to prevent the tendon from going completely slack during a simulation. The tendon attaches the muscle fibers to the bones in the musculoskeletal model, so the force in the tendon is the force generated by the muscle model as a whole, so $F^{MT}(t) = F^T(t)$.

The computation of fiber velocity proceeds as follows. The width of each muscle, kept constant as $w = l_0^M \sin \alpha_0$, and the current fiber length $l^M(t)$ are used to calculate the pennation angle

$$\alpha(t) = \begin{cases} 0, & l^M(t) = 0 \text{ or } w/l^M(t) \leq 0 \\ \sin^{-1}(w/l^M(t)), & 0 < w/l^M(t) < 1. \\ \pi/2, & w/l^M(t) \geq 1 \end{cases}$$

The active force in the muscle fibers is computed as

$$F_a(t) = a(t) f_l(l^M(t)) = a(t) F_0^M \bar{f}_l(\bar{l}^M(t)), \text{ where}$$

$$\bar{f}_l(\bar{l}^M(t)) = e^{-(\bar{l}^M(t)-1)^2/\gamma}$$

is a Gaussian function representing the normalized active force-length relationship for all

muscles in our musculoskeletal model. The passive force in the muscle fibers is

computed as $F^{PE}(t) = F_0^M \bar{F}^{PE}(\bar{l}^M(t))$, where

$$\bar{F}^{PE}(\bar{l}^M(t)) = \begin{cases} 1 + \frac{k^{PE}}{\varepsilon_0^M} (\bar{l}^M - (1 + \varepsilon_0^M)), & \bar{l}^M > 1 + \varepsilon_0^M \\ \frac{e^{k^{PE}(\bar{l}^M - 1)/\varepsilon_0^M}}{e^{k^{PE}}}, & \bar{l}^M \leq 1 + \varepsilon_0^M \end{cases}$$

is a function representing the normalized passive force-length relationship for all muscles

in our musculoskeletal model. This function is affine for large forces (the first case in the

above equation) and is otherwise exponential (the second case in the equation). The total

effect of the normalized passive and active force-length relationships yields a function

that increases, levels off, decreases slightly, and then increases rapidly. All muscles in

our musculoskeletal model of the body have this same normalized force-length property.

It has been shown that the shape of the total force-length curve for different muscles in

the body are not identical (Gareis et al., 1992): while some muscles have a slight decrease

after the level portion of the total force-length curve, others have a significant decrease,

and some have an increase instead of a decrease. Therefore, our model is an approximation of the total force-length relationships of muscles in real humans.

The force in the contractile element is then calculated as

$$F^{CE}(t) = \frac{F^T(t)}{\cos \alpha(t)} - F^{PE}(t).$$

The distinction between the force in the contractile element and the active force

calculated above is that the active force does not include the effects of the force-velocity

relationship of the muscle fibers. We calculate this force-velocity scale factor as

$$\bar{F}_V^M \left(\bar{l}^M(t + \Delta t) \right) = \frac{F^{CE}(t)}{F_a(t)}.$$

\bar{F}_V^M is an invertible function representing the normalized force-velocity relationship. The normalized fiber velocity is calculated as

$$\bar{l}^M(t + \Delta t) = \left(\bar{F}_V^M \right)^{-1} \left(\frac{F^{CE}(t)}{F_a(t)} \right),$$

where, for any normalized force f ,

$$(\bar{F}_V^M)^{-1}(f) = \begin{cases} \left(1 + \frac{1}{A_f}\right)f - 1, & f < 0 \\ \frac{f-1}{1 + \frac{f}{A_f}}, & 0 \leq f < 1 \\ \frac{(f-1)(\bar{F}_{len}^M - 1)}{\left(2 + \frac{2}{A_f}\right)(\bar{F}_{len}^M - f)}, & 1 \leq f < 0.95\bar{F}_{len}^M \\ \frac{10(\bar{F}_{len}^M - 1)}{\left(1 + \frac{1}{A_f}\right)\bar{F}_{len}^M} \left(-18.05\bar{F}_{len}^M + 18 + \frac{20f(\bar{F}_{len}^M - 1)}{\bar{F}_{len}^M}\right), & 0.95\bar{F}_{len}^M \leq f \end{cases}.$$

The exact implementation of $(\bar{F}_V^M)^{-1}$ is slightly different to adjust for possible numerical issues as described below.

The fiber velocity (un-normalized) is

$$\dot{l}^M(t + \Delta t) = V_{max} \bar{\dot{l}}^M(t + \Delta t),$$

where $V_{max} = (5 + 5a)l_0^M$.

Note that $\bar{\dot{l}}^M$ is different from $\dot{\bar{l}}^M$. $\bar{\dot{l}}^M$, which we calculated above, is the fiber velocity

normalized (i.e., divided) by V_{max} , while $\dot{\bar{l}}^M$ is the fiber velocity normalized (i.e.,

divided) by l_0^M :

$$\dot{\bar{l}}^M = \frac{d}{dt}[\bar{l}^M] = \frac{d}{dt}\left[\frac{l^M}{l_0^M}\right] = \frac{1}{l_0^M} \frac{d}{dt}[l^M] = \frac{\dot{l}^M}{l_0^M}.$$

The original normalized force-velocity relationship is

$$\bar{F}_V^M(v) = \begin{cases} \frac{v+1}{1+\frac{1}{A_f}}, & v < -1 \\ \frac{v+1}{1-\frac{v}{A_f}}, & -1 \leq v < 0 \\ \frac{\left(2+\frac{2}{A_f}\right)v\bar{F}_{len}^M + \bar{F}_{len}^M - 1}{\left(2+\frac{2}{A_f}\right)v + \bar{F}_{len}^M - 1}, & 0 \leq v < \frac{10(\bar{F}_{len}^M - 1)(0.95\bar{F}_{len}^M - 1)}{\left(1+\frac{1}{A_f}\right)\bar{F}_{len}^M} \\ \frac{\frac{\bar{F}_{len}^M}{20(\bar{F}_{len}^M - 1)} \left(\frac{\left(1+\frac{1}{A_f}\right)\bar{F}_{len}^M v}{10(\bar{F}_{len}^M - 1)} + 18.05\bar{F}_{len}^M - 18 \right)}{\frac{10(\bar{F}_{len}^M - 1)(0.95\bar{F}_{len}^M - 1)}{\left(1+\frac{1}{A_f}\right)\bar{F}_{len}^M}}, & \frac{10(\bar{F}_{len}^M - 1)(0.95\bar{F}_{len}^M - 1)}{\left(1+\frac{1}{A_f}\right)\bar{F}_{len}^M} \leq v \end{cases}.$$

This function was inverted to obtain the expressions for $(\bar{F}_V^M)^{-1}$ above, but in the implementation, some additional constants $\xi = 0.05$ (a passive damping factor for the force-velocity relationship) and $\varepsilon = 10^{-6}$ are incorporated to prevent possible numerical errors. The complete implementation of the inverted force-velocity relationship is as follows.

$$(\bar{F}_V^M)^{-1}(F^{CE}, F_a) = \begin{cases} \frac{F^{CE}}{\varepsilon} \left(\frac{\varepsilon - F_a}{F_a + \frac{\varepsilon}{A_f} + \xi} + \frac{F_a}{F_a + \xi} \right) - \frac{F_a}{F_a + \xi}, & F^{CE} < 0 \\ \frac{F^{CE} - F_a}{F_a + \frac{F^{CE}}{A_f} + \xi}, & 0 \leq F^{CE} < F_a \\ \frac{F^{CE} - F_a}{\left(2 + \frac{2}{A_f}\right)(F_a \bar{F}_{len}^M - F^{CE})} + \xi, & F_a \leq F^{CE} < 0.95 F_a \bar{F}_{len}^M \\ f_{v0} + \frac{F^{CE} - 0.95 F_a \bar{F}_{len}^M}{\varepsilon F_a \bar{F}_{len}^M} (f_{v1} - f_{v0}), & 0.95 F_a \bar{F}_{len}^M \leq F^{CE} \end{cases},$$

where

$$f_{v0} = \frac{0.95 F_a \bar{F}_{len}^M - F_a}{\left(2 + \frac{2}{A_f}\right) 0.05 F_a \bar{F}_{len}^M + \xi}$$

and

$$f_{v1} = \frac{(0.95 + \varepsilon) F_a \bar{F}_{len}^M - F_a}{\left(2 + \frac{2}{A_f}\right) (0.05 - \varepsilon) F_a \bar{F}_{len}^M + \xi}.$$

The curves representing the active force-length, passive force-length, and force-velocity properties of fibers and the elasticity of tendon are shown in Figure 1 below.

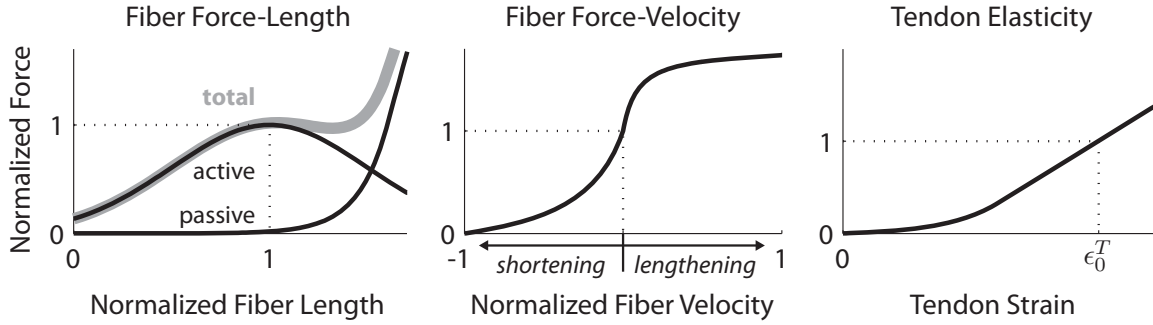


Figure 1. Curves representing the intrinsic properties of the Thelen, 2003 muscle model used in our study. The curves shown here are “normalized”, meaning that for a specific muscle in our musculoskeletal model, the actual curves would be obtained by scaling the normalized curves vertically by maximum isometric force (F_0^M) and scaling the curves in the three graphs horizontally by optimal fiber length (l_0^M), maximum contraction velocity (V_{max}), and tendon slack length (l_s^T), respectively, and then shifting the tendon elasticity curve horizontally by the tendon slack length (l_s^T). The active force-length curve and force-velocity curve would have a smaller vertical range when the muscle is less than fully activated ($a < 1$).

References

- Thelen DG (2003) Adjustment of muscle mechanics model parameters to simulate dynamic contractions in older adults. *J Biomech Eng* 125: 70-77.
- Gareis H, Solomonow M, Baratta R, Best R, D'Ambrosia R (1992) The isometric length-force models of nine different skeletal muscles. *J Biomech* 25: 903-916.